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Summary of Curve Smoothing Technology Development

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Abstract:

With the continuous development of computer technology, the application of free curve in the design and manufacture of modern industrial products is more and more widely used.However, due to errors in the design and measurement process, the smoothness of the curve often can not meet the design and manufacturing requirements.The smoothness of the curve affects not only the appearance of the product, but also the difficulty of manufacturing technology and the mechanical properties of the products. It is an important symbol to evaluate the quality of products. Discussed the interpolation of the curve and the approximate construction method, the smoothing criterion and the smoothing method and introduced the relationship between smooth curve and smooth surface.

***Keywords* —Curve, fairing, Geometric modeling**

1. INTRODUCTION

Computer Aided Design (CAD) and Computer Aided Manufacturing (CAM) technology originated in the aviation industry, due to the complex aircraft shape, with a large number of free-form curves and surfaces, therefore, CAD / CAM technology from the very beginning with the freeform surface modeling technology are closely linked. Curved surface modeling technology was established by Coons, Bezier and other masters in the 1960s [1]. After years of research and development, curve shape of surface formed by NURBS curve characteristics of parameterized design and implicit algebraic curves and said the two types of method as the main body, by means of interpolation, approximation, fitting for the skeleton geometry theory system [2].

Along with the development of the industry, because of the NURBS method can accurately said second rule curve surface, through the power factor is easy to control and implementation, and can directly promote such outstanding advantages in thinking space, in 1991, the international organization for standardization (ISO) define the NURBS as the geometry of the industrial products only mathematical method.Since then, NURBS method has become the most important foundation in the trend of curve surface modeling technology [3].

2. CURVE INTERPOLATIONANDAPPROXIMATION CONSTRUCTION MRTHOD

Interpolation occurs first in the field of numerical analysis and is usually interpreted to generate new data through known data.In a curved structure, it is given that some data points generate a smooth curve.Approximations also generate new data through known data.The difference between the two is that the curve produced by the interpolation method passes through all the points given, calling these points data points. The curves generated by the approximation method do not necessarily go through all the points. These points are called control points[4].

2.1.The Bezier Curve

In the early 1960s, Bezier and De Casteljau proposed the same curve construction method by different algorithms--Bezier curve [5]. Bezier method is polynomial interpolation and approximation.Now we have a given point $P_i(0 \le i \le n)$ and the number is $n+1$,if given $n+1$ weighted coefficient, meet the following conditions $\sum_{i=0}^{n} W_i = 1$, the point $Q = \sum_{i=0}^{n} W_i P_i$, can think of it as a linear combination or the point of weighted average. If W_i is a function of parameter t, that's what we're talking about $W_i = W_i(t)$, $t \in [a,b]$, usually, the value of t is $[0,1]$, and then $P(t)\sum_{i=0}^{n} W_i(t) P_i$, it represents a parametric curve, and at this moment $W_i(t)$ is base function or blending function, the polygon that connects the control point P_i is called the control polygon. To construct a smooth curve, Bezier chose Bernstein polynomial as the basis function, it is $B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$, in this function, n is the number of points. This equation $\sum_{i=0}^{n} B_i^n(t) = 1$ guarantees the affine invariance of the curve. It can be seen that $B_i^n(t)$ is a polynomial and its frequency is n. Bezier curve interpolation P_0、P_n, they are Attracted by other control points. This Bezier curve approximates the curve. You can control the shape of the curve by moving these points, or you can add and remove points to change the shape.

You can also construct interpolated Bezier curves. Now we have a given point $Q_i(0 □ i □ n)$, choose t and its number is $n+1$ which make $P(t_i) = Q_i$, in the form of a matrix is expressed as:

$$\begin{pmatrix} B_0^n(t_0) & B_1^n(t_0) & \cdots & B_n^n(t_0) \\ B_i^n(t_1) & B_1^n(t_1) & \cdots & B_n^n(t_1) \\ \vdots & \vdots & \ddots & \vdots \\ B_0^n(t_n) & B_1^n(t_n) & \cdots & B_n^n(t_n) \end{pmatrix} \begin{pmatrix} P_0 \\ P_1 \\ \vdots \\ P_n \end{pmatrix} = \begin{pmatrix} Q_0 \\ Q_1 \\ \vdots \\ Q_n \end{pmatrix} \quad (1)$$

The equation (1)is said as $MP = Q$, Let's multiply both sides of this equation by M^{-1} and obtained $P = M^{-1}Q$. Then, the drawing of the curve is obtained by the obtained P_i, and the Bezier curve of interpolation Q_i is obtained.

2.2.The B-spline Curve

The b-spline construction method of the curve was proposed in the 1940s, and the breakthrough was developed in the 1970s, such as Rich Riesenfeld, De Boor et al[5]. B-spline technology has overcome the disadvantages of Bezier technology, has the advantages of local control, and has high order continuity. Similarly, the B spline curve is the approximation curve.

The b-spline curve has the same expression as the Bezier curve, but the selection principle of the base function is different.

The b-spline curve introduces the node vector, which is the parameter value interval of the curve.Suppose you have a known point $P_i(0 \le i \le n)$ and its number is $n+1$, the curve $P(t) = \sum_{i=0}^{n} W_{k,i}(t) P_i$, $t \in [a,b]$ is called a K-order B-spline curve, if the basis function $W_{k,i}(t)$ is a K degree polynomial, $1 \le k \le n$, $P(t)$ is a piecewise polynomial function and it has a continuous derivative at the junction until $k-1$.

While $k=n$, the B-spline curve degenerates into a Bezier curve, so the Bezier curve is a special case of the B-spline curve. The structure of the base function $W_{k,i}(t)$ adopts the cox-de Boor recursive algorithm [6].

The B-spline curve introduces the node vector, which is the parameter value range of the curve.The point vector is a non-decreasing sequence of real Numbers $(t_0, t_1, \cdots, t_m)$, where t_i is called a node. The distribution of nodes produces different B-splines and the uniform B-spline has more flexible control, that is, control points can be manipulated, or nodes can be moved or inserted. However, a B-spline curve can not represent a conic curve because the function of the conic curve is a rational polynomial. Therefore, rational B-spline is introduced. Generally, use point homogeneous coordinate to understand rational B-spline. Three-dimensional point $P_i(x_i, y_i, z_i)$ can use homogeneous coordinates as $P_i^{wi}(w_i x_i, w_i y_i, w_i z_i, w_i)$. Substituting the homogeneous coordinates into the B-spline curve's parametric equation:

$$P^w(t)=\sum_{i=0}^{n} W_{k,i}(t) P_i^{w_i} = \sum_{i=0}^{n} W_{k,i}(t) \begin{pmatrix} w_i x_i \\ w_i y_i \\ w_i z_i \\ w_i \end{pmatrix} = \begin{pmatrix} \sum_{i=0}^{n} W_{k,i}(t) w_i x_i \\ \sum_{i=0}^{n} W_{k,i}(t) w_i y_i \\ \sum_{i=0}^{n} W_{k,i}(t) w_i z_i \\ \sum_{i=0}^{n} W_{k,i}(t) w_i \end{pmatrix}$$

The first three components with the fourth component into three-dimensional form is:

$$P(t)=\begin{pmatrix} \sum_{i=0}^{n} W_{k,i}(t) w_i x_i / \sum_{i=0}^{n} W_{k,i}(t) w_i \\ \sum_{i=0}^{n} W_{k,i}(t) w_i y_i / \sum_{i=0}^{n} W_{k,i}(t) w_i \\ \sum_{i=0}^{n} W_{k,i}(t) w_i z_i / \sum_{i=0}^{n} W_{k,i}(t) w_i \end{pmatrix} = \sum_{i=0}^{n} \frac{w_i W_{k,i}(t)}{\sum_{j=0}^{n} w_j W_{k,j}(t)} p_i = \sum_{i=0}^{n} R_{k,i}(t) P_i$$

You can see the rational polynomial of basis function $R_{k,i}(t)$. Rational B-spline can accurately represent the conic curve [5].

3. CURFACESMOOTHING

The smoothness of the curve mainly includes two aspects: the smooth and the smooth method [3][7].

3.1.Curve Smoothing Criterion

It is generally considered that the following conditions should be included in the smoothness criterion:second order continuous curve; there are no singularities or excess inflection points;the curvature change is relatively uniform (some curves are not only due to the sharp increase or decrease of curvature, but also the curve is not smooth);strain energy is small.

3.2.Curve Smoothing Method and Analysis

The smoothing method is to make a certain mathematical treatment of not only the smooth curve to achieve the smooth (or the smoothness).

In the existing light smoothing method, according to the light smoothing criterion and the number of modified value points, it is divided into local smoothing method [8-10] and the whole smoothing method [11-13].

Local smoothing method, such as round rate method, legal persons and other are modified method for choosing the base samples, is under the assumption that most values are good or better, on the basis of the minority not only along the area of the type value points (known as "bad points") amended pick out one by one.Usually, this kind of method has strong local modification ability and fast calculation speed, but it is not ideal when the "bad point" has more time.The common feature of local smoothing method is that the curve can be adjusted locally according to the specific requirements of the curve design. The disadvantage is that the algorithm can only adjust one control vertex at a time.Local algorithm suitable for some simple curve fairing, complex curve to the large amount of data using local fairing method often need to repeatedly adjust curve, not only the computation, and difficult to guarantee the overall effect of the curve.

The whole smoothing method, such as the least square method, energy method and springback method, is the minimization problem of the transformation of the smooth problem into the objective function, and the basic idea is the same.The target function contains the deviation of the type value point and the weighted average of the two parts.

Generally, this kind of method is ideal for overall smoothing effect, but the calculation is large and the convergence speed is slow.The energy method is a widely used curvilinear smoothing method. The energy method takes into account the minor criterion of strain energy, and the uniform criterion of curvature change is not considered.In addition, using the energy method to smooth the light, no matter what the geometrical shape of the smooth, the smooth effect always tends to the linear change, so in some cases the smooth effect is not satisfactory.

At present, most CAD/CAM systems adopt the parameter B-spline curve technology, and the most common method in engineering practice is to interpolate the curve by importing or creating the type value point.The interpolation curve is strictly based on the given value point, and the premise of the interpolation algorithm is accurate.However, due to the measurement error and other reasons, it is impossible to get the exact type point.In addition, for some have more than one area of the curve, not only because of the design itself, some areas can not according to rule of fairing fairing, otherwise it will affect the product's design intent.

4. THE RELATIONSHIP BETWEEN SMOOTH SURFCE AND SMOOTH SURFACE

The process of reverse modeling is usually composed of the point structure, the surface is connected by the transition surface, and the surface reconstruction is completed. Therefore, the smooth design of the curve is the basis of smooth surface design.

Class A surface is A proper noun in the field of automobile, first put forward by the French dassault company, it should not only conform to the requirements of the continuity of mathematics, and compliance requirements, to meet the requirements of aerodynamics and light control points should be evenly arranged on the surface [14].It is difficult to reconstruct the curvature continuous surface with the point cloud to construct the curvature continuous surface.Automotive covering parts A level surface to achieve the curvature, must be based on the curvature of curve reconstruction surface curvature change even no redundant inflection point and the curvature error within 0.5 mm hard to reach A level surface.

Surface smoothness is an important criterion for evaluating A-grade surface, and the surface smoothing methods include curvature detection method, energy analysis, wavelet analysis, genetic algorithm smoothing and illumination detection.For A single surface, the A class surface requires it to be concave.

Requirements in the industrial area A-class surface must meet the following requirements:

(1) the order of the surface in the two directions should be between 3 and 7 times, and the maximum should not be greater than 9 times.
(2) the adjacent surface film satisfies G^2 continuous, and the special requirement satisfies G^3.
(3) large feature curved plates are single convex.

5. CONCLUSIONS

The development of curve smoothing technology provides strong support for modern manufacturing industry.The modeling technique is based on the traditional curve interpolation, approximation, fitting and extension to various optical smoothing techniques.The CAD industry has also rapidly applied new technologies and theories to industry applications, and the importance of visible light technology to modern industry.Looking at the development of geometric modeling technology, it is the key to promote the whole CAGD research to find better curve smoothing technology.

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